

vSLAM: Visual Simultaneous Location and Mapping

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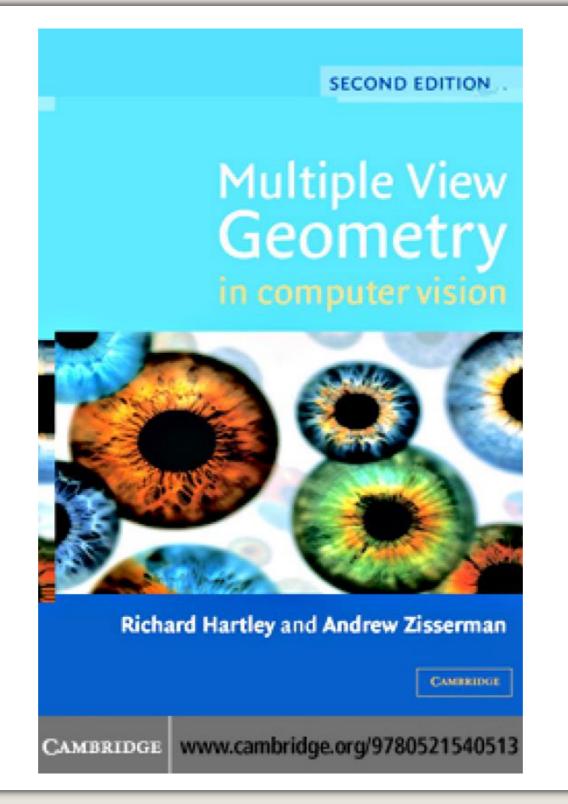
- Explaining building blocks for 3D reconstructions
 - Two-view geometry in more detail
 - Triangulation
 - Bundle adjustment
 - PnP
 - Loop closure with visual location recognition
- Putting all the pieces together
 - Hierarchical SfM







THE Reference for Most of this Lecture







Recap: Camera Matrix

Pinhole camera model (calibrated vs. uncalibrated case)

$$\mathbf{P} = \mathbf{K} \begin{bmatrix} \mathbf{R}, \mathbf{t} \end{bmatrix} \qquad \mathbf{K} = \begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

- Principal point **p**
 - Intersection of principal axis with image plane
 - Principal axis: line through camera centre orthogonal to image plane
- Camera center (aka. centre of projection, pinhole) in world coordinates?

$$\mathbf{C} = \begin{pmatrix} -\mathbf{R}^T \mathbf{t} \\ 1 \end{pmatrix}$$

- Affine camera model
 - Camera center at infinity
 - Parallel projection





Recap: Two-View Geometry

- Two-view geometry
 - Fundamental matrix $\mathbf{x'}^T \mathbf{F} \mathbf{x} = 0$
 - Degree of freedoms?
 - Essential matrix $\tilde{\mathbf{x}'}^T \mathbf{E}\tilde{\mathbf{x}} = 0$ with $\mathbf{E} = \mathbf{K}^T \mathbf{F} \mathbf{K} = [\mathbf{t}]_{\times} \mathbf{R}$
 - Degree of freedoms?
 - 5-point algorithm
 - Needs to solve a 10th degree univariate polynomial
 - Provides 10 solutions (counting multiplicities; some of them complex)
 - See also "Five-Point Motion Estimation Made Easy" by Hongdong Li and Richard Hartley
- Estimation of fundamental or essential
 - Find keypoints and extract feature descriptors
 - Putative correspondences by matching feature descriptors
 - RANSAC loop for geometric verification



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Example: Pre & Post RANSAC

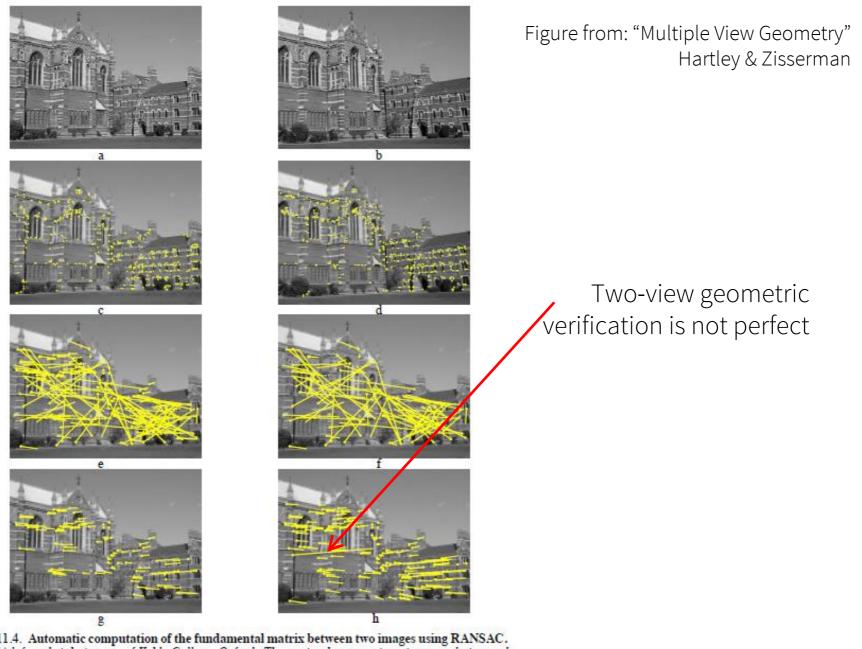


Fig. 11.4. Automatic computation of the fundamental matrix between two images using RANSAC. (a) (b) left and right images of Keble College, Oxford. The motion between views is a translation and rotation. The images are 640×480 pixels. (c) (d) detected corners superimposed on the images. There are approximately 500 corners on each image. The following results are superimposed on the left image: (e) 188 putative matches shown by the line linking corners, note the clear mismatches; (f) outliers – 89 of the putative matches. (g) inliers – 99 correspondences consistent with the estimated F; (h) final set of 157 correspondences after guided matching and MLE. There are still a few mismatches evident, e.g. the long line on the left.







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Decomposing the Essential Matrix

- Assume the correct essential has been found
- Goal: decompose essential into rotation and translation
- Problem: decomposition is not unique: 4 solutions exist
 - With: $svd(\mathbf{E}) = \mathbf{U} diag(1, 1, 0) \mathbf{V}^T$

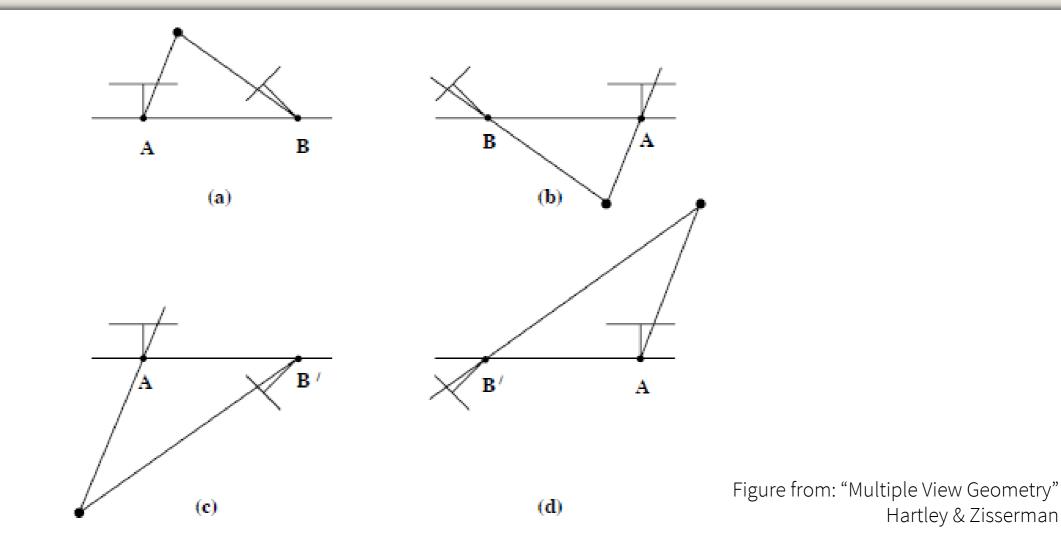
$$\begin{bmatrix} \mathbf{t} \end{bmatrix}_{\times} = \pm \mathbf{U} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{U}^T \qquad \mathbf{R} = \mathbf{U} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{V}^T \text{ or } \mathbf{R} = \mathbf{U} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{V}^T$$

See also MVG sec. 9.6.2 (2nd edition)





Decomposing the Essential Matrix



- Interpretation
 - baseline reversal
 - Rotation of one camera 180° about baseline
- Points are in front of camera only in one solution



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Two-View Geometry in Practice

- Be aware of planar scenes
 - Homography explains point correspondences $\mathbf{x}' = \mathbf{H}\mathbf{x}$
 - Let's pick a random point \mathbf{y}' in 2nd view
 - Consider line spanned by \mathbf{x}' and $\mathbf{y}' : \mathbf{l}' = [\mathbf{y}']_{\times} \mathbf{x}' = [\mathbf{y}']_{\times} \mathbf{H}\mathbf{x}$
 - Obviously, \mathbf{x}' lies on this line: $0 = \mathbf{x}'^T \mathbf{l}' = \mathbf{x}'^T [\mathbf{y}']_{\times} \mathbf{H} \mathbf{x}$
- Less of a problem for essential matrices
 - But you'll get two equally valid but different solutions...
- Ill-conditioned motions
 - Pure rotations: reveals no 3D structure
 - Forward motions



 $=\mathbf{F}$



Figure from: "Multiple View Geometry" Hartley & Zisserman

- Rotation-translation ambiguity: translation vs. rotation around an axis far away
 - Severe problem for nearly planar scenes with small depth variation
 - Especially important for narrow field of view (like on mobile phones)



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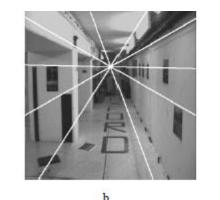
Triangulation

- Assume known
 - Camera poses (for at least two frames)
 - Image correspondences
- Shoot rays through image points and intersect in 3D
 - Rays won't intersect due to image noise
 - Minimizing meaningful reprojection error is non-trivial
 - Example: 2-view triangulation

$$\min_{\hat{\mathbf{x}}, \hat{\mathbf{x}}'} \left\| \frac{1}{x_3} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \frac{1}{\hat{x}_3} \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} \right\|_2^2 + \left\| \frac{1}{x_3'} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} - \frac{1}{\hat{x}_3'} \begin{pmatrix} \hat{x}_1' \\ \hat{x}_2' \end{pmatrix} \right\|_2^2$$

s.t. $\hat{\mathbf{x}}'^T \mathbf{F} \hat{\mathbf{x}} = 0$

- Leads to roots of 6th degree univariate polynomial
- Quiz: Are there 3D points which can't be triangulated from two views?
 - Yes: points on baseline (ie. Points which project onto epipoles)





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Figure from: "Multiple View Geometry" Hartley & Zisserman



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Triangulation

- Assume known
 - Camera poses (for at least two frames)
 - Image correspondences
- Direct Linear Transform (DLT)
 - Simple method, minimizes algebraic error
 - Eliminate scale factor (= projective depth)

 $\mathbf{x}_{f} \cong \mathbf{P}_{f} \mathbf{X} \Leftrightarrow \lambda_{f} \mathbf{x}_{f} = \mathbf{P}_{f} \mathbf{X}$

 $[\mathbf{x}_f]_{\times}\mathbf{x}_f = \mathbf{0}_{3\times 1} = [\mathbf{x}_f]_{\times}\mathbf{P}_f\mathbf{X}$

- Stack measurements from all images and solve with SVD $\min_{\mathbf{X} \in \mathbb{R}^4} \left\| \left[\bigcup_f \left[\mathbf{x}_f \right]_{\times} \mathbf{P}_f \right] \mathbf{X} \right\|_2^2$
- What about points at infinity?





Triangulation

- Depth uncertainty of triangulated 3D points mostly depends on angle between intersected rays
 - Small angle → inaccurate triangulation

- Small baseline → small angle
- Large baseline → large angle?
 - Not always true! Example?
 - Forward motions...





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Bundle Adjustment

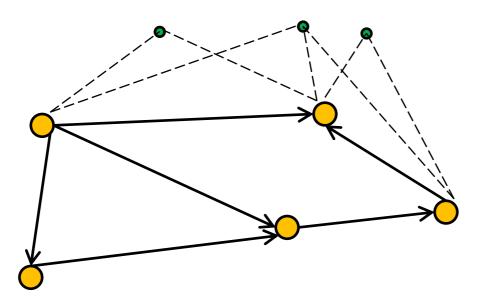
- We know how to perform 2-view reconstruction
- Assume we have initial guess of 3D reconstruction
- Goal: refine a meaningful geometric error
 - Reprojection error
 - Cycle consistency when camera sees same points again after making a loop





Point-Pose-Graph

- Conceptual representation of SfM
 - Vertices: camera poses & 3D points
 - Edges
 - Edges between camera vertices if estimate of relative pose is available (eg. from essential matrix)
 - Edges between camera and 3D point if point has been seen in this camera (= measurements)

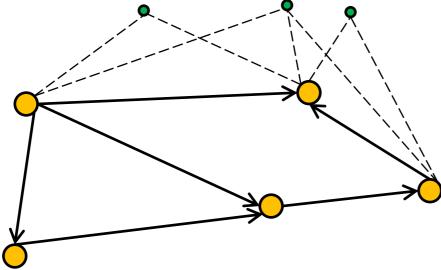






Bundle Adjustment

- Unknowns
 - 3D coordinates of points & camera poses
- Data evidence
 - 2D feature point correspondences
- Initial guess available
 - Decompose pairwise essentials + three view verified
- Refine initial guess by minimizing reprojection error while adhering to cycle constraints
 - Modern BA frameworks phrase optimization problem as optimization over point-pose graph
 - " "g2o: A General Framework for Graph Optimization" Kümmerle et.al. [ICRA11]







Bundle Adjustment: Parameterization of Unknowns

- Rotation matrices
 - Euler angles (avoid them if possible)
 - Unit-quaternions
 - Angle-axis & exponential-map
- 3D points (aka. Landmarks in robotics community)
 - Inhomogeneous coordinates (x,y,z)
 - Problem: points at infinity (or 'sufficiently' far away)
 - Homogeneous coordinates (x,y,z,w)
 - Problem: arbitrary scale per point leads to rank deficiency in Hessian
 - Inverse depth parameterization of point relative to a camera (eg. the one which has observed the point first)
 - No problems with points at infinity
 - Reprojection error becomes 'more linear' \rightarrow Important for filtering based SLAM systems
 - "Inverse Depth Parameterization for Monocular SLAM" Civera, Davison, Montiel [Trans. On Robotics 08]





Bundle Adjustment: Numerical Details

non-linear robust LS with residuals $\mathbf{r}_{ij} = \mathbf{x}_{ij} - f(\mathbf{X}_i, \mathbf{R}_j, \mathbf{K}_j)$

- Linearize residual and compute update direction: $\Delta \mathbf{x}^{t} = \arg \min_{\Delta \mathbf{x}} \|\mathbf{r}^{t-1} + \mathbf{J} \Delta \mathbf{x}\|_{2}^{2}$ Gauss-Newton approximation of Hessian: $\mathbf{H} \approx \mathbf{J}^{T} \mathbf{J} = \begin{bmatrix} \mathbf{H}_{XX} & \mathbf{H}_{XC} \\ \mathbf{H}_{XC}^{T} & \mathbf{H}_{CC} \end{bmatrix}$
- Choose 'smart' parameterization for rotations & robust cost function (not L2)
- Computation of update direction: Gauss-Newton with Schur-complement trick

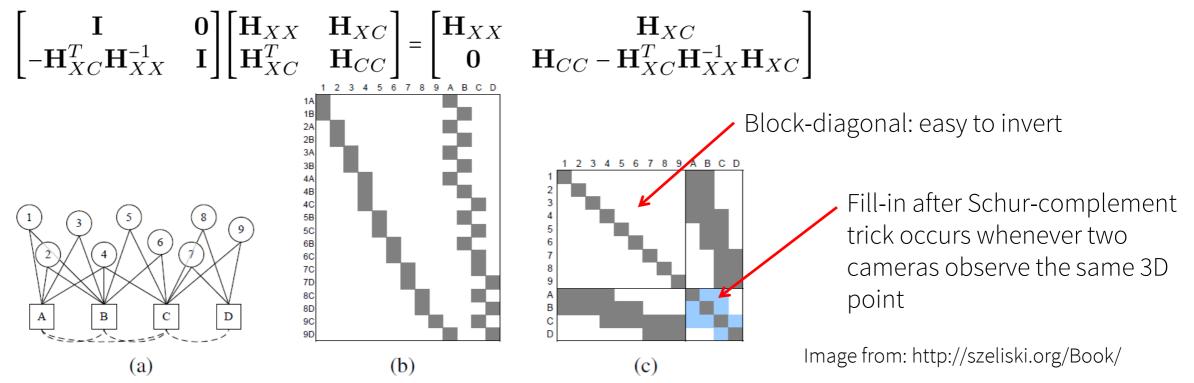


Figure 7.9 (a) Bipartite graph for a toy structure from motion problem and (b) its associated Jacobian J and (c) Hessian A. Numbers indicate 3D points and letters indicate cameras. The dashed arcs and light blue squares indicate the fill-in that occurs when the structure (point) variables are eliminated.





Bundle Adjustment: Gauge Freedom

- Choice of global coordinate system is arbitrary
 - Often fixed to first camera $\mathbf{P}_1 = [\mathbf{I}_3, \mathbf{0}_{3 \times 1}]$
 - 1st camera has no error
 - Introduces bias since error is not distributed evenly across all cameras
- Relative BA
 - Idea: Let's not select and designate a single global coordinate system
 - Instead: Choose multiple coordinate systems to express variables
 - Express 3D points relative to camera which first observed point
 - Relative transformations between coordinate systems allow to transform 3D points to other coordinate system
 - Pro
 - Error is more evenly spread
 - Loop closures can be handled better
 - Con
 - Jacobian matrices of BA become denser due to chaining relative transformations
 - *Relative Bundle Adjustment Based on Trifocal Constraints" Steffen, Frahm, Förstner [ECCV10]





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PnP Motivation: Sequential SfM, vSLAM

- SLAM: Simultaneous Location And Mapping
 - Terminology used in robotics
 - vSLAM: visual SLAM based entirely on images
 - Known as sequential SfM in computer vision
- Sequential SfM (aka. Incremental SfM)
 - Initialize structure and motion from two views
 - For each new image
 - Compute camera pose given 3D structure from previous iteration (PnP problem)
 - Refine camera poses (new & previous ones) and structure with BA
 - 'Densify' structure by triangulating new 3D points
- vSLAM = "sequential SfM in realtime" with video stream from camera





PnP Problem

Perspective n-Point camera pose computation

- Compute camera pose from n given 3D-2D point correspondences
- Calibrated case: How many correspondences are minimally required?
 - 3 (be aware: up to four solutions)
 - P3P: "Review and Analysis of Solutions to the Three Point Perspective Pose Estimation Problem" Haralick et.al. [IJCV94]
 - OpenCV methods: solvePNP(...) and solvePnPRansac(...)
- Efficiency of PnP makes sequential SfM so attractive
 - RANSAC efficiency largely depends on minimal sample size!





P3P again boils down to solving polynomial equations...

$$Y^{2} + Z^{2} - YZp - a'^{2} = 0$$

$$Z^{2} + X^{2} - XZq - b'^{2} = 0$$

$$X^{2} + Y^{2} - XYr - c'^{2} = 0.$$

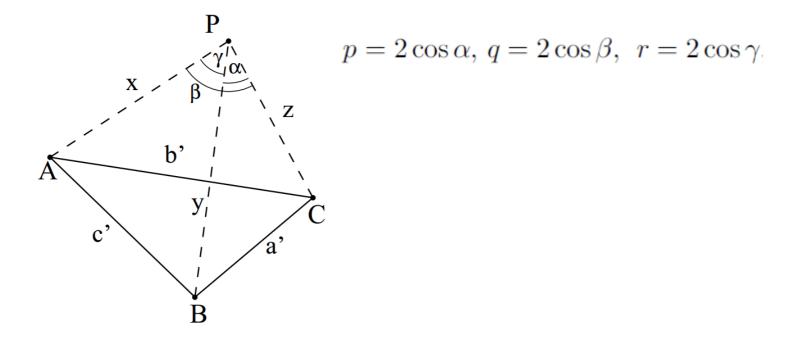


 Figure from: "Complete Solution Classification for the Perspective-Three-Point Problem" Gao et.al. [PAMI03]





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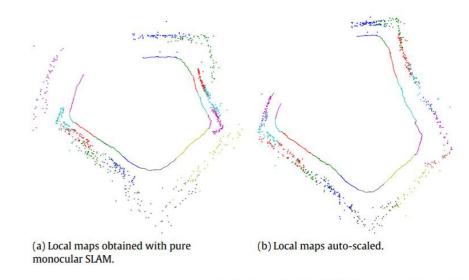


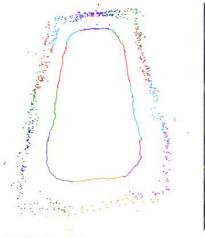




Loop Closure and Scale Drift

- Loop closure problem
 - Accumulation of error in sequential SfM or SLAM leads to gaps in cycles
 - 3D structure might not overlap when closing a loop
 - Visual SLAM and sequential SfM especially suffer from scale drift
- Loop detection
 - Detect which parts should overlap
 - Leads to cycles in pose-graph
 - Cycles stabilize BA







(c) After loop closure.

(d) Aerial view of the courtyard.

"A comparison of loop closing techniques in monocular SLAM"

Williams et.al. [RAS09]

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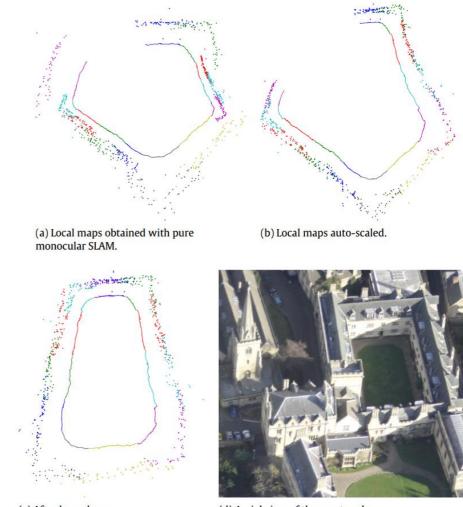
Loop Detection

- Appearance based approaches most popular
 - Similar techniques used for image retrieval



"Scalable recognition with vocabulary tree" Nister & Stewenius [CVPR06]

- Extract discriminative feature descriptors of keyframes
 - SIFT, SURF, etc.
- Store descriptors in efficient search data structure
 - Inverted index, vocabulary tree, ...
- Issue a query with descriptors of query image and verify if any of top-K results is geometrically consistent



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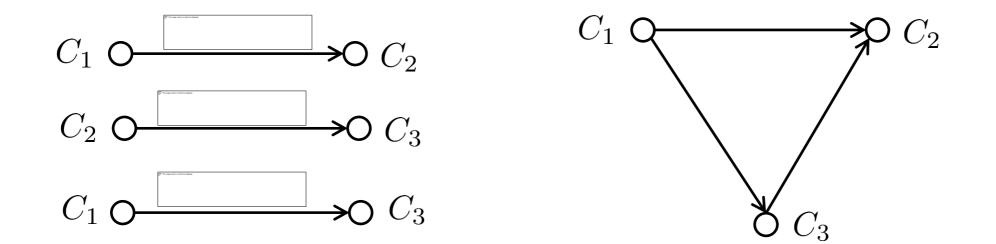






Hierarchical Structure-from-Motion (SfM)

- For each pair of images
 - Perform 2-view reconstruction \rightarrow set of two view reconstructions
- Triplet generation
 - Assemble pairwise reconstructions which share a common camera into triplets







Hierarchical Structure-from-Motion (SfM)

- Increase robustness by three-view verification (loop consistency)
 - Cycle consistent relative poses
 - Remove spurious matches which survived two-view verification (eg. due to repetitive texture)
 - Slight complication: translations from pairwise reconstructions are only known up to scale
 - Choose arbitrary scale between first image pair, eg. $\|\mathbf{t}_{12}\|_2 = 1$
 - ${}^{\blacksquare}$ 3D points jointly seen in views 1,2, and 3 provide scale for $~t_{13}~t_{23}$
- Register verified triplets (using shared edges)
 - Again pay attention to different scale in neighboring triplets
- Merge sub-reconstructions
 - Sprinkle BA steps in-between





Hierarchical Structure-from-Motion (SfM)

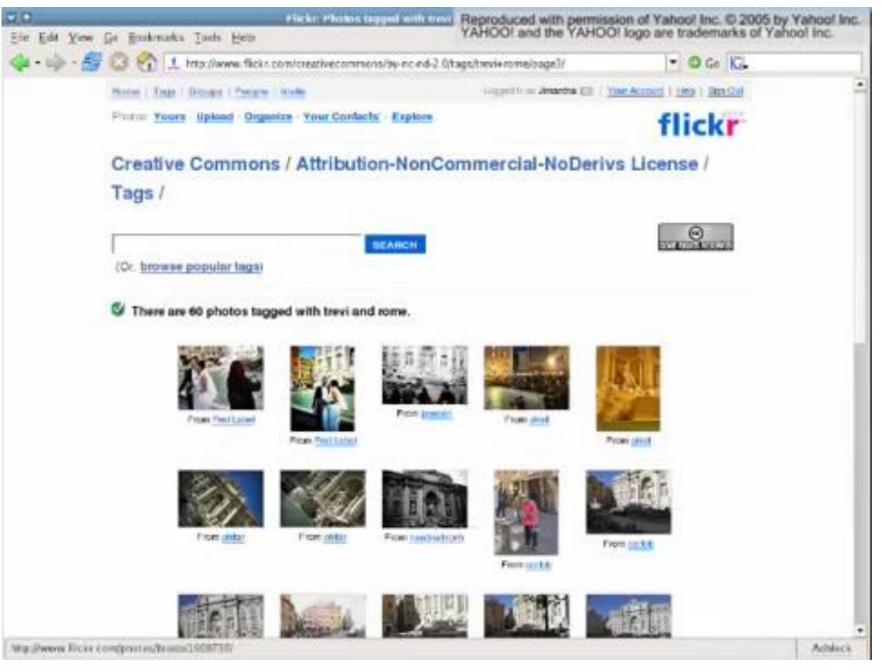
Challenges

- Generation of high-quality correspondences
- Handling thousands of images: Avoid pairwise matching of images
- Large scale optimization problem with many local minima
- Repetitive structures
 - windows and building facades are highly repetitive...





Results



- Photo Tourism [2006]
 - http://phototour.cs.washington.edu/





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vSLAM





Visual SLAM

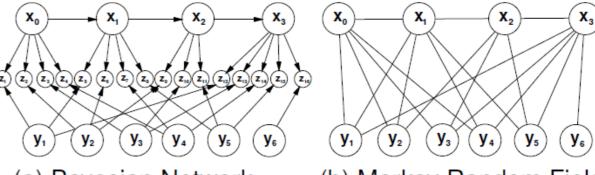
- Stream of temporally ordered images
- Simultaneously compute 3D map and camera pose w.r.t. map
- Two main approaches
 - Filtering
 - Key-frame based





Filtering vs. Key-Frames

- Recall SfM point-pose graph
 - Bipartite graph
 - 3D landmarks vs. camera poses
 - Filtering: marginalize over previous camera poses
 - State: 3D landmarks + current camera pose
 - Key-Frame BA: keep subset of frames as keyframes
 - State: 3D landmarks + camera poses for all key frames
 - Nowadays preferred
 - BA can refine state in a thread separate from tracking component
 - "Parallel Tracking and Mapping for Small AR Workspaces" Klein & Murray [ISMAR07]



(a) Bayesian Network

(b) Markov Random Field

"Real-Time Monocular SLAM: Why Filter?" Strasdat et.al. [ICRA10]



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Information to Keep Track of

State

- Camera poses of keyframes
- 3D coordinates of reconstructed points
- Data evidence
 - 2D locations of detected keypoints
 - Descriptors of keypoints
 - Additional data: timestamps, IMU data, ...
- Bookkeeping: Data association
 - Which 2D keypoints correspond to a certain 3D point?
 - Sometimes replicated multiple times for faster queries
 - Which keyframes have observed a given 3D point?
 - Which 3D point corresponds to a given 2D keypoint?
 - Some systems keep track of multiple descriptors per 3D point
 - Handles appearance changes of 3D points
 - Also helpful for relocalization





Keyframe-Based SLAM: Operation modes

vSLAM system has 3 main modes of operation

Bootstrapping

- Compute an initial 3D map
- Mostly based on concepts from two-view geometry

Normal mode

- Assumes a 3D map is available and incremental camera motion
- Track points and use PnP for camera pose estimation

Recovery mode

- Assumes a 3D map is available, but tracking failed: no incremental camera motion anymore
- Relocalize camera pose w.r.t. previously reconstructed map



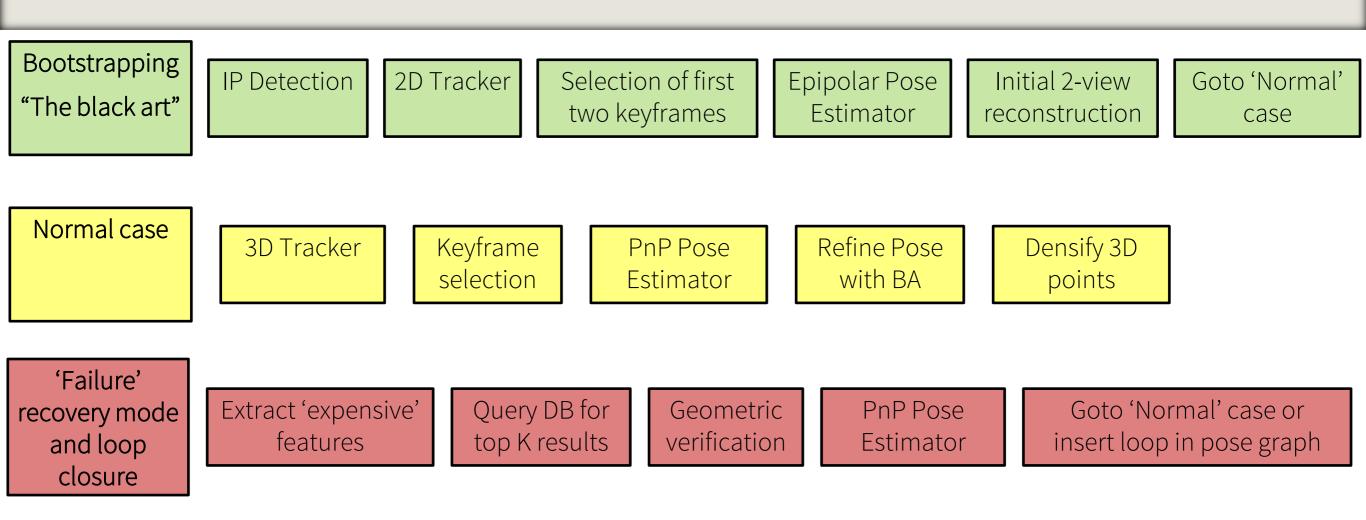


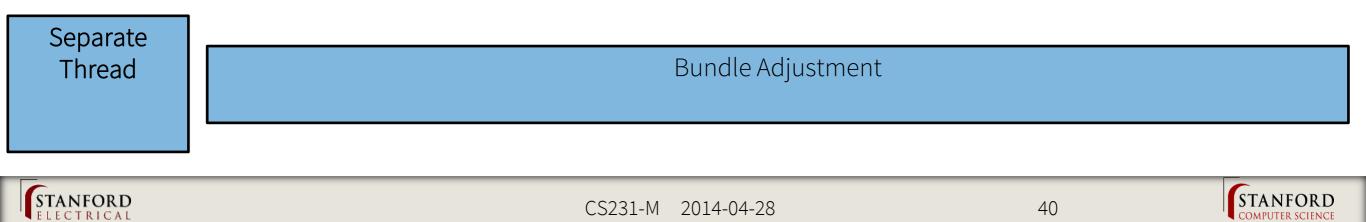
System Components

- Bootstrapping
 - Initial 3D map generation
- 3D tracker and PnP pose estimator
 - Processes incoming frames as quickly as possible
- Relocalization
 - Recovering from tracking failure
 - Can also be used for loop closure detection
- Mapping data structure
 - Point-pose graph
- Bundle adjustment
 - Runs in separate thread and refines estimates
 - Accesses mapping data structure









vSLAM Results



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Monocular SLAM

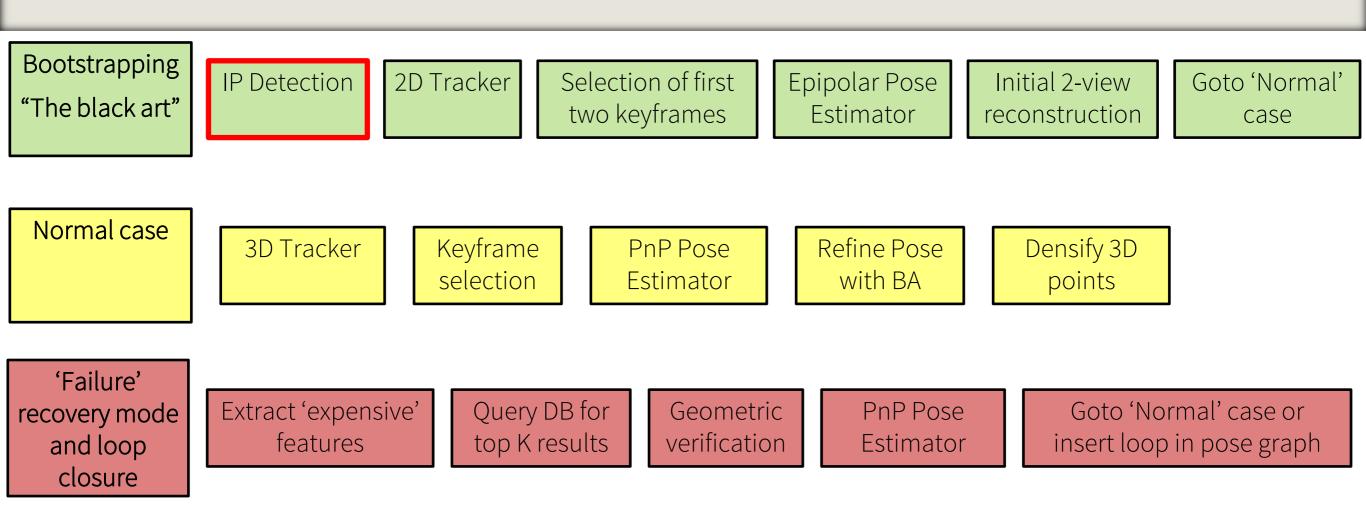
local browsing motion

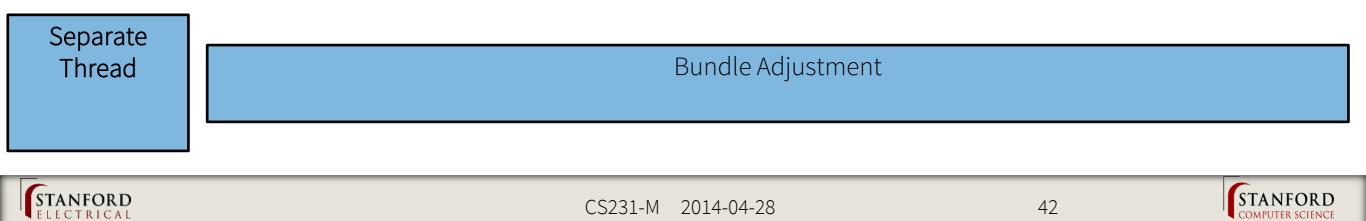
"Double Window Optimisation for Constant Time Visual SLAM" Strasdat et.al. [ICCV11]



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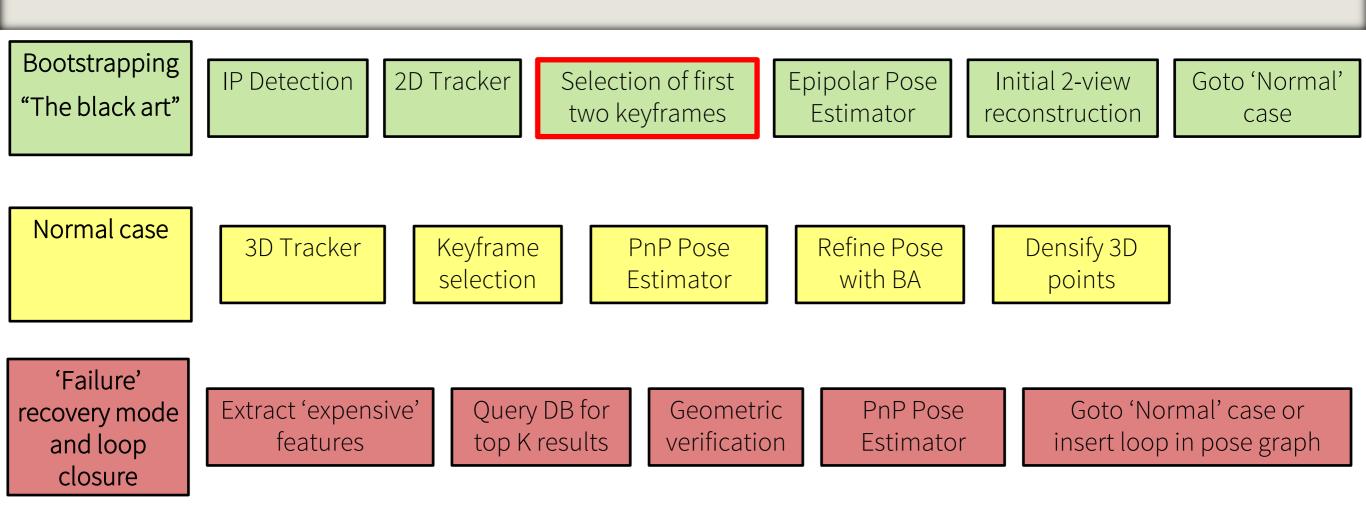
IP Detection

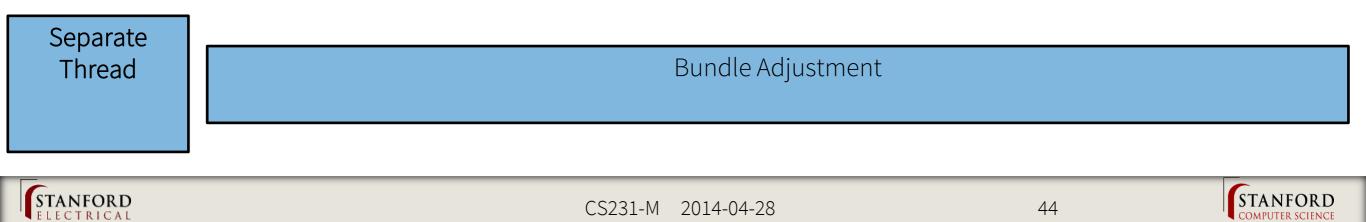


- Avoid clusters of interest points
 - RANSAC estimates suffer when many IPs are close together
- Roughly uniformly distributed IP
 - Introduce grid
 - Avoid imbalanced number of IPs in grid cells
- Be aware of complexity of IP detector and descriptor
 - SIFT is powerful, but expensive to compute
- Many options available
 - IP Detectors: FAST, Harris corner, Scalespace extrema (SIFT), MSER, ...
 - Descriptors: image patch, BRISK, SIFT, ...









Initial Selection of Two Keyframes

Avoid "non-parallax views"

- Pure rotation of camera
 - In practice: "pure" depends on [unknown] depth of points
 - Motion of points at infinity will always appear as due to pure rotation
- Low-parallax views
 - Small translations and forward motion
- Avoid planar scenes
 - Fundamental matrix is ill-defined for planar scenes
 - Essential can be estimated, but care must be taken!



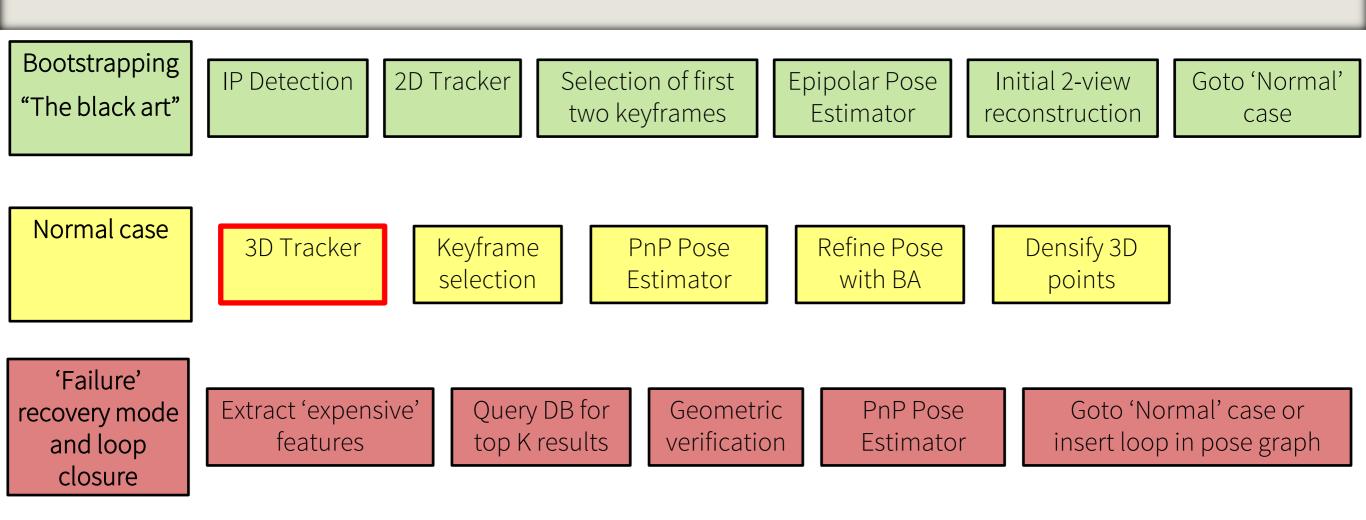


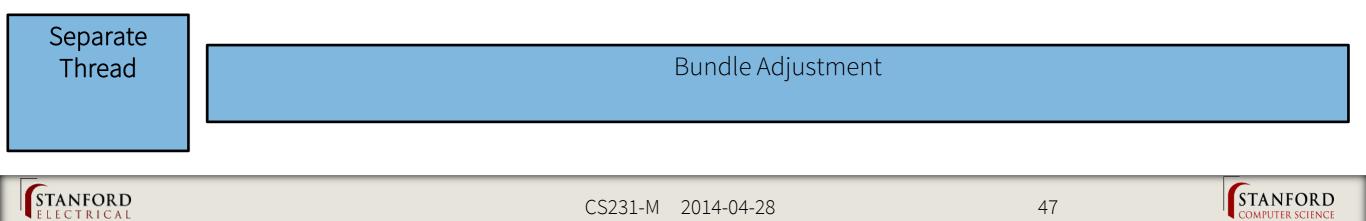
Initial Selection of Two Keyframes

- How to avoid theses cases without knowing 3D structure and camera poses?
- Check for planar scene
 - Can correspondences be explained with homography?
 - If yes, raise red flag
- Check for sufficiently large parallax
 - Compensate for displacements due to camera rotation
 - Can be done very efficiently if gyroscope is available
 - Are remaining displacements sufficiently large?
 - If yes, good for triangulation
 - Compensation for camera rotation
 - Decompose essential into rotation and translation
 - Apply rotation as homography to image measurements (similar to stereo rectification)
 - Remaining displacement between feature points is due to translation









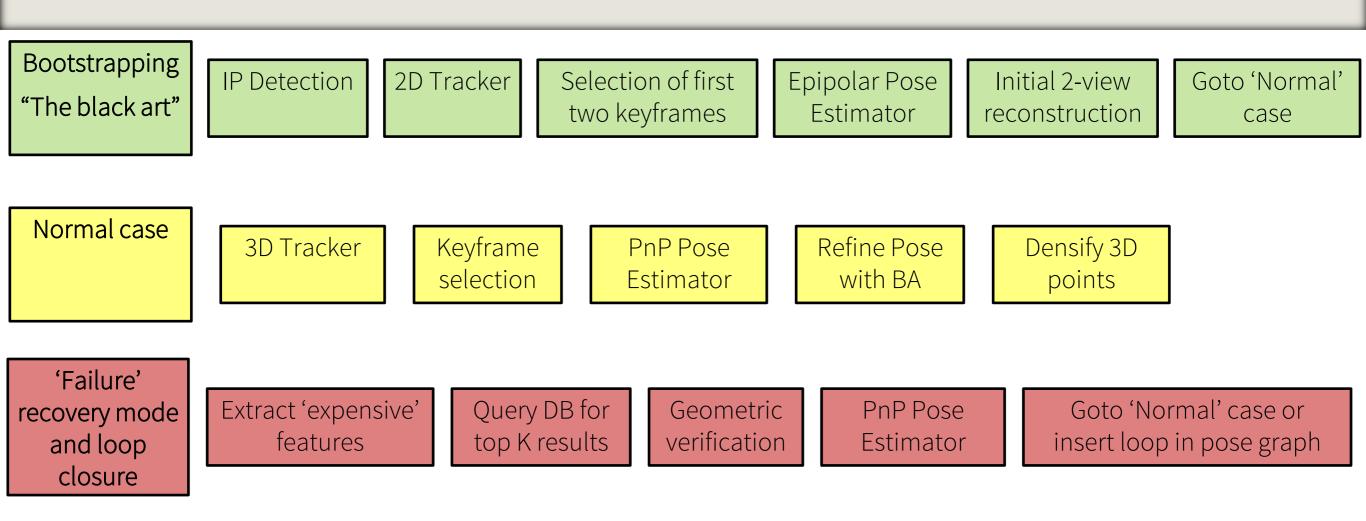
Active Search

- Also known as Guided Search
- Avoid searching naïvely for IP and matching descriptors
- Setting: Incremental camera motion and known depth of 3D points
 - Good initial guess available where to expect corresponding point
 - Can also include motion model of camera (eg. constant velocity)
 - Or IMU measurements
 - For example: patch-based KLT tracker (Kanade-Lucas-Tomasi)
 - See also lecture on Wednesday

Active Search and PnP makes vSLAM efficient!









Bundle Adjustment

- Bundle adjustment is a big topic on its own
- Recent approaches
 - "Double Window Optimisation for Constant Time Visual SLAM" Strasdat et.al.
 [ICCV11]
 - Split BA objective into two terms
 - Cycle consistency of loops
 - Reprojection error
 - Minimize within window of recent frames
 - "Towards Linear-time Incremental Structure from Motion" Changchang Wu [3DV13]
 - Carefully designed sequential SfM system
 - Conjugate gradient with early termination instead of Cholesky





Ideas for Class Projects

BA

- Implementation of conjugate gradient based BA approach with double window optimization
- Exploit IMU data
 - Gyroscope, accelerometer, compass
 - Motion field for feature tracking
 - Accelerometer provides measurements in metric units
 - Very noisy measurements
 - Estimation of absolute scale still possible
- Self-calibration App (aka. auto-calibration)
 - Assumptions about intrinsics lead to constraint for each frame on camera matrices
 - Examples: Square pixels, constant but unknown focal length, ...
- Line-based SfM
 - Lines are strong cues for pose estimation
 - Especially in indoor scenes
- Dense reconstructions on the phone







