

RANSAC: RANdom Sampling And Consensus

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CS231-M 2014-04-30



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The Need for RANSAC

- Why do I need RANSAC? I know robust statistics!
 - "Robust Statistics" Huber [1981]
 - M-estimator, L-estimator, R-estimators, ...
 - Least Median of Squares (LMedS), ...
- Breakdown point of an estimator
 - "Proportion of incorrect observations ... an estimator can handle before giving an incorrect ... result" [Wikipedia]
- Robust estimators can achieve breakdown point of 50%
 - For example: median
- Usually a non-linear, non-convex optimization problem needs to be solved





The Need for RANSAC

Problems

- Estimators for more complex entities (eg. homographies, essential matrices, ...)?
- Inlier ratio of computer vision data can be lower than 50%

Hough Transform

- Excellent candidate for handling high-outlier regimes
- Can only handle models with very few parameters (roughly 3)
- RANSAC is a good solution for models with slightly larger number of parameters
 - Roughly up to 10 parameters (depending on inlier ratio)





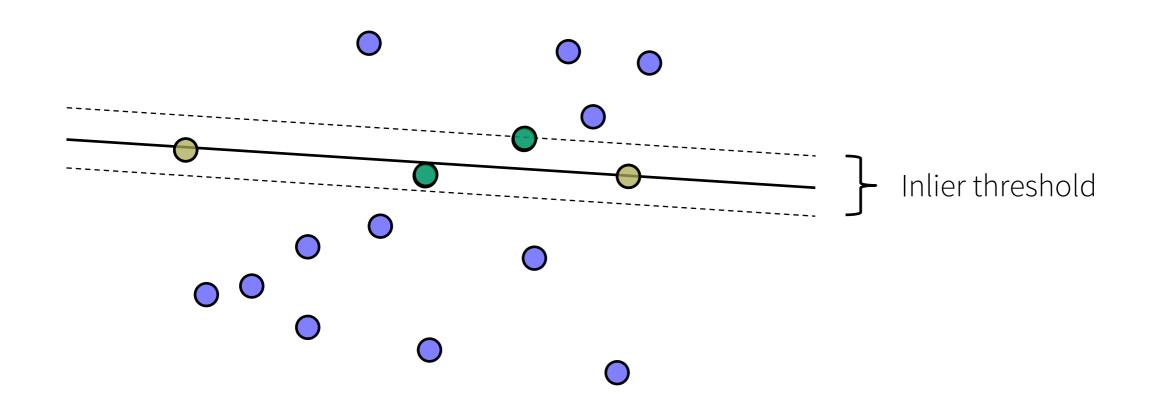
RANSAC [Fischler & Bolles 81]

- Hypothesize-and-verify framework
 - Sample hypothesis and verify with data
- Assumptions
 - Outliers provide inconsistent (ie. random) votes for models
 - There are sufficiently many inliers to detect a correct model
- Hypothesis generation
 - Sample subset of data points and fit model parameters to this subset
 - Plain RANSAC: sample points uniformly at random
- Verification on all remaining data points





Algorithm Outline

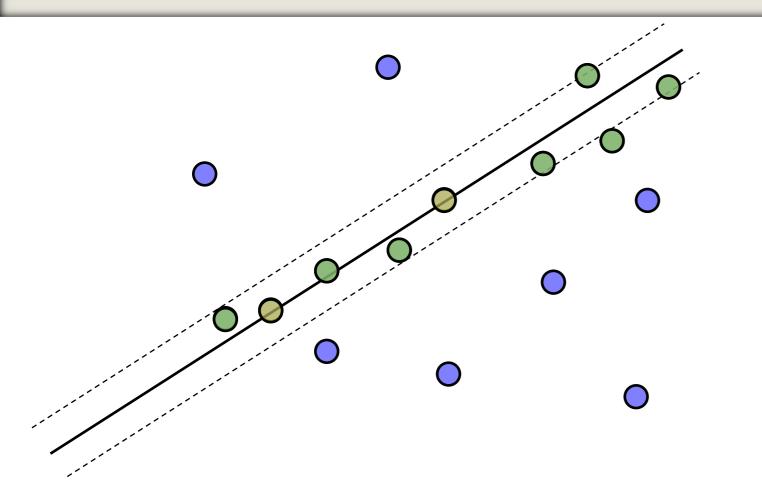


- 1. Select random sample of minimum required size to fit model parameters
- 2. Compute a putative model from sample set
- 3. Verification stage: Compute the set of inliers to this model from whole data set
- 4. Check if current hypothesis is better than any other of the previously verified
- 5. Repeat 1-4





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Number of Iterations

- Probability of selecting an inlier given by inlier ratio p_{inlier}
- Sample size *s*
- Confidence value for having sampled at least one all-inlier sample *P*
- Number of iterations k
- Let's put all of this together: $1 P \ge ((1 p_{inlier}^s)^k)$

		F	p = 0.99; pi	roportion	ofoutlier	S	
S	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Probability of having selected at least one outlier in each of the k trials

$$k \ge \frac{\log(1-P)}{\log(1-p_{\text{inlier}}^s)}$$





RANSAC Parameters

- How to find inlier ratio?
 - Provide lower bound for initialization and recompute when new best hypothesis has been found
- Scale of inlier noise
- Confidence for having sampled at least one all-inlier sample





- Scale of inlier noise (for inlier-outlier threshold) needs to be specified
- Correct model is not generated with user-defined confidence
- Estimated model might be inaccurate
- Degenerate cases not handled
- Can be sped up considerably
 - Better hypothesis generation
 - Faster verification schemes
- Multiple models
 - Model selection
 - Interesting problem, but not covered in remainder





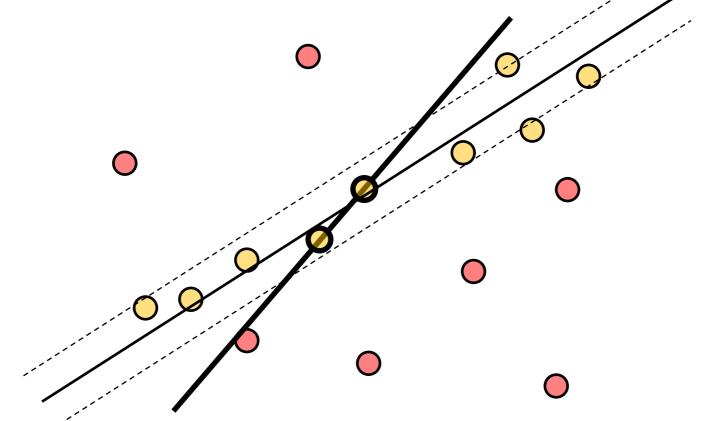
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Noisy Inliers

Problem: not every all-inlier-sample provides a good solution



Sampling more than one all-inlier-set might be necessary!

In practice, solution often found only after roughly $k = \left(\frac{1}{p_{\text{inlier}}}\right)^{k}$ iterations

Simple calculation
$$k \ge \frac{\log(1-P)}{\log(1-p_{\text{inlier}}^s)}$$
 is inaccurate





sample size

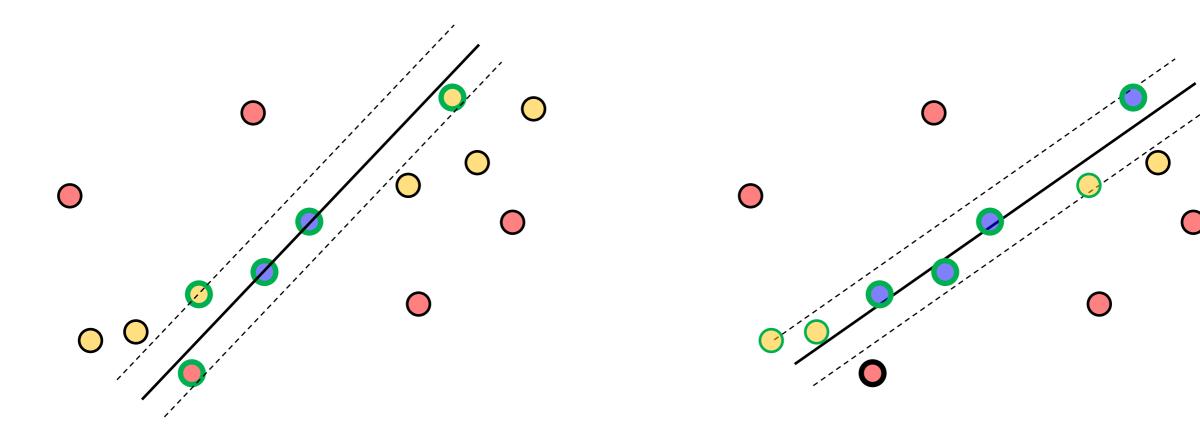
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Increase Accuracy of Estimated Models

- Lo-RANSAC
 - Run inner RANSAC loop with non-minimal sample size to refine hypothesis of minimal sample size
 - Locally Optimized RANSAC "Chum, Matas, Kittler [DAGM03]



MLESAC

- Fit model by max likelihood rather than max inlier count
- MLESAC: A new robust estimator with application to estimating image geometry" Torr & Zisserman [1996]

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Handling Degenerate Cases

- "Two-view geometry estimation unaffected by a dominant plane" Chum et.al. [CVPR05]
 - Estimate fundamental
 - If successful try to fit homography to triplet of 7-cardinalty MSS
 - If homography can be found run plane-and-parallax fundamental estimation
 - 2 points off the plane need to get fundamental from known homography
 - 2-pt RANSAC over outliers of homography
 - else non-planar case

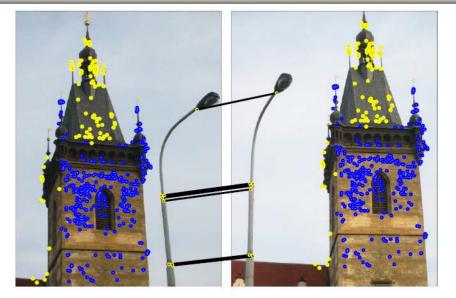


Figure 1: The LAMPPOST scene with 97% of correct tentative correspondences lying in or near a dominant plane. In 100 runs, RANSAC fails to find a single inlier on the lamp 83 times; in the remaining 17, no more than 4 out of the 10 correspondences on the lamppost are found. Points on the lamppost are far from the dominant plane and therefore critically influence the precision of epipolar geometry and egomotion estimation. The DEGENSAC algorithm, with the same computational complexity as RANSAC, found the 10 lamppost inliers in all runs. Corresponding points lying in the dominant plane are dark, off-the-plane points are light, and the points on the lamp are highlighted by line segments.

- Other approaches for making RANSAC robust w.r.t. degeneracies
 - "RANSAC for (quasi-)degenerate data (QDEGSAC)" Frahm & Pollefeys [CVPR06]







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Hypothesis Generation

- Trade-off between exploration and exploitation
 - Previously verified hypothesis tell us something about inlier set
 - Still, we should avoid narrowing our search too quickly
- Especially important for multi-model case
 - Eg. estimation of multiple planes in a scene
 - Points on other planes act as outliers to plane under consideration





PROSAC

- "Matching with PROSAC progressive sample consensus" Chum & Matas [CVPR05]
- Use of a-priori knowledge
 - Confidence of a matching pair (eg. based on descriptor matching distance)
- PROSAC: Favor high-quality matches while sampling points for minimal sample
 - Sort correspondences according to matching score
 - Consider progressively larger subsets of putative correspondences
 - Note: draws the **same** samples as RANSAC would, just in different order
- Pro
 - Can decrease the number of required hypothesis considerably
- Contra
 - Performance gain depends on data
 - Practical observation: high-confidence matches appear often appear in clusters on same spatial structure
 - Degenerate configurations...





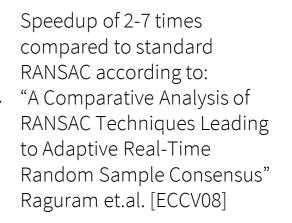
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Verification

- Phrase hypothesis verification in sequential testing framework
 - Subsample remaining data and verify on this subset
 - If inlier ratio is sufficiently low: terminate verification
- Several papers have been published
 - Threshold determined based on Td,d tests
 - "Randomized RANSAC with Td,d test" Matas, Chum [IVC04]
 - Bail-Out test based on hyper-geometric distribution
 - "An effective bail-out test for RANSAC consensus scoring" Capel [BMVC05]
 - Wald's Sequential Probability Ratio Test (WaldSAC)
 - "Optimal randomized RANSAC" Chum & Matas [PAMI07]





Preemptive RANSAC

- "Preemptive RANSAC for live structure and motion estimation" Nister [ICCV03]
- Find a good estimate within a fixed time budget (eg. in a vSLAM system)

Idea

- Generate fixed number of hypothesis
- Verify all of them in parallel
 - Breadth-first verification scheme
 - Verify all hypothesis on a subset of the data
 - Prune unpromising hypothesis and retain promising ones
 - Verify on increasingly larger subsets, followed by pruning step





ARRSAC

- Adaptive Real-Time RANSAC
- Carefully designed combination of previous RANSAC approaches
- Achieves considerable speed-ups while still providing correct solution
- "A Comparative Analysis of RANSAC Techniques Leading to Adaptive Real-Time Random Sample Consensus" Raguram et.al.
 [ECCV08]

Table 1. Evaluation results for ten selected real image pairs. The images show variation over a range of inlier ratios and number of correspondences. It can be observed from the above results that the ARRSAC approach produces significant computational speed-ups, while simultaneously providing accurate robust estimation in real-time. In practice, the ARRSAC technique stays well within the time budget, with estimation speeds ranging between 55-350 Hz. It can be seen from the table that the number of hypotheses evaluated by ARRSAC is always less than preemptive RANSAC. In addition, the correct epipolar geometry is always recovered.

		RANSAC	$T_{d,d}$	Bail- out	Wald	PROSAC	Pre- emptive	ARRSAC
A : $\epsilon = 0.83, N = 1322$	I k vpm spd-up	884 112 1322 1.0	884 156 531 1.9	885 118 361 2.6	889 148 594 2.2	885 5 1322 12.9	933 500 396 0.4	$ \begin{array}{r} 1099 \\ 6 \\ 434 \\ 37.3 \\ \end{array} $
$\mathbf{B}: (\boldsymbol{\varepsilon} = 0.7, N = 795)$	I k vpm spd-up	514 242 795 1.0	512 329 149 2.4	514 242 65 4.1	517 283 132 3.9	514 43 795 7.7	527 500 374 0.3	559 47 218 11.1
C : $(\varepsilon = 0.65, N = 2162)$	I k vpm spd-up	$1344 \\ 208 \\ 2162 \\ 1.0$	1342 947 59 1.9	1344 208 67 5.5	1345 211 52 5.8	1344 17 2162 15.2	$1372 \\ 500 \\ 400 \\ 0.9$	$ \begin{array}{r} 1413 \\ 17 \\ 430 \\ 29.1 \end{array} $
D : $(\varepsilon = 0.33, N = 1500)$	I k vpm spd-up	483 15506 1500 1.0	22 1.9	484 15509 37 5.7	484 15514 28 6.2	484 201 1500 104.2	350 500 398 90.6	$491 \\ 241 \\ 59 \\ 305.7$
$\mathbf{E}: (\boldsymbol{\varepsilon} = 0.44, N = 420)$	I k vpm spd-up	$174 \\ 2714 \\ 420 \\ 1.0$	$ \begin{array}{r} 174 \\ 4889 \\ 10 \\ 2.0 \\ \end{array} $	173 2719 24 4.4	177 2716 13 5.7	$177 \\ 439 \\ 420 \\ 5.3$	$134 \\ 500 \\ 305 \\ 11.5$	183 499 70 20.2
F : (ϵ = 0.43, N = 1325)	I k vpm spd-up	$557 \\ 4353 \\ 1325 \\ 1.0$	557 11753 5 2.1	557 4366 31 4.7	$557 \\ 4359 \\ 24 \\ 5.3$	557 288 1325 6.9	488 500 396 12.3	582 329 182 15.1
G : (ε = 0.56, N = 2875)	I k vpm spd-up	$ \begin{array}{r} 1537 \\ 432 \\ 2875 \\ 1.0 \\ \end{array} $	1524 628 20 2.2	1539 437 24 3.1	1537 432 31 4.2	$1537 \\ 30 \\ 2875 \\ 16.6$	$1539 \\ 500 \\ 400 \\ 1.3$	$ \begin{array}{r} 1616 \\ 33 \\ 456 \\ 47.2 \end{array} $
H : (ε = 0.67, N = 1986)	I k vpm spd-up	1249 335 1986 1.0	1247 1153 29 2.4	1251 351 41 4.2	1252 347 32 4.9	1250 17 1986 22.2	$1237 \\ 500 \\ 400 \\ 1.1$	$ \begin{array}{r} 1330 \\ 18 \\ 467 \\ 60.9 \end{array} $
I : $(\varepsilon = 0.76, N = 1508)$	I k vpm spd-up	$945 \\ 498 \\ 1508 \\ 1.0$	934 1644 43 1.9	948 505 31 2.7	951 502 33 2.5	$949 \\ 58 \\ 1508 \\ 12.3$	960 500 398 1.8	$ \begin{array}{r} 1149 \\ 65 \\ 277 \\ 45.1 \end{array} $
J : ($\epsilon = 0.61, N = 1521$)	I k vpm spd-up	872 362 1521 1.0	872 1167 39 2.2	873 381 28 6.1	873 372 25 6.3	872 40 1521 9.8	870 500 398 1.9	$927 \\ 44 \\ 338 \\ 26.1$



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Further Evaluation and Comparisons

"Performance Evaluation of RANSAC Family" Choi et.al. [BMVC09]

Inlier Ratio	0.300	0.400	0.500	0.600	0.700	0.800	0.900	Mag. of Noise	0.250	0.500	1.000	2.000	4.000
LMedS	27.320	6.461	1.321	1.356	1.408	1.379	1.509	LMedS	1.379	1.372	1.295	1.256	1.228
RANSAC	1.305	1.323	1.326	1.330	1.390	1.415	1.483	RANSAC	1.385	1.380	1.315	1.282	1.259
MSAC	1.229	1.266	1.284	1.337	1.373	1.415	1.535	MSAC	1.389	1.319	1.306	1.246	1.193
MLESAC	1.248	1.269	1.289	1.316	1.358	1.410	1.446	MLESAC	1.364	1.341	1.309	1.262	1.218
LO-RANSAC	1.245	1.229	1.229	1.221	1.229	1.253	1.255	LO-RANSAC	1.219	1.224	1.202	1.203	1.222
R-RANSAC.T	1.317	1.323	1.304	1.341	1.394	1.401	1.475	R-RANSAC.T	1.396	1.363	1.286	1.232	1.180
R-RANSAC.S	15.210	1.848	1.131	1.229	1.291	1.370	1.389	R-RANSAC.S	1.253	1.325	1.413	1.583	1.618
FH' MAPSAC	7.708	2.036	1.647	1.484	1.463	1.490	1.526	FH'MAPSAC	1.458	1.557	1.513	1.395	1.406
AMLESAC	2.051	1.477	1.452	1.529	1.517	1.526	1.575	AMLESAC	1.479	1.499	1.942	1.192	1.164
GASAC	28.640	7.370	1.108	1.077	1.100	1.120	1.147	GASAC	1.098	1.100	1.102	1.115	1.124
pbM-estimator	1.023	1.034	1.209	1.255	1.286	1.291	1.355	pbM-estimator	1.258	1.272	1.264	1.296	1.306
uMLESAC	5.246	1.382	1.402	1.383	1.398	1.433	1.489	uMLESAC	1.361	1.232	1.069	1.025	1.051
RANSAC*	47.010	13.920	3.031	1.688	1.386	1.255	1.187	RANSAC*	1.389	1.379	1.409	1.440	1.552
MLESAC*	50.110	10.240	2.839	1.694	1.352	1.235	1.145	MLESAC*	1.357	1.326	1.330	1.388	1.468





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RANSAC Conclusion

Many different 'flavours'

Still an active research area

	Google	RANSAC	
	Scholar	Albut 4,450 results (0.04 lec)	
	Articles Case law My library	(HTML] Respiratory complex III dysfunction in humans and the use of yeast as a model organism to study mitochondrial myopathy and associated diseases B Meunier, <u>N Fisher</u> , S Ransac, JP Mazat Biochimica et Biophysica, 2013 - Elsevier Abstract The bc 1 complex or complex III is a central component of the aerobic respiratory chain in prokaryotic and eukaryotic organisms. It catalyzes the oxidation of quinols and the reduction of cytochrome c, establishing a proton motive force used to synthesize Cited by 3 Related articles All 4 versions Cite Save	[HTML] from sciencedirect.com Find it@Stanford
<	Any time Bince 2014 Since 2013 Since 2019 Custom range	[HTML] <u>Mitochondrial energetic metabolism—some general principles</u> JP Mazat, S Ransac, M Heiske, A Devin IUBMB life, 2013 - Wiley Online Library Summary In nonphotosynthetic organisms, mitochondria are the power plant of the cell, emphasizing their great potentiality for adenosine triphosphate (ATP) synthesis from the redox span between nutrients and oxygen. Also of great importance is their role in the Cited by 3 Related articles All 6 versions Web of Science: 1 Cite Save	[HTML] from wiley.com Find it@Stanford
	Sort by relevance Sort by date ✓ include patents ✓ include citations	[CITATION] <u>Outliers Elimination Based Ransac for Fundamental Matrix Estimation</u> S Yang, B Li - Virtual Reality and Visualization (ICVRV), 2013, 2013 - ieeexplore.ieee.org Abstract—To accelerate the RANSAC process for fundamental matrix estimation, two special modifications about RANSAC are proposed. Firstly, in the verification stage, not the correspondences are used to verify the hypothesis but the singular values of estimated Related articles Cite Save	Find it@Stanford
	≌ Create alert	Image Based 6-DOF Camera Pose Estimation with Weighted RANSAC 3D J Wetzel - Pattern Recognition, 2013 - Springer Abstract In this work an approach for image based 6-DOF pose estimation, with respect to a given 3D point cloud model, is presented. We use 3D annotated training views of the model from which we extract natural 2D features, which can be matched to the query image 2D Related articles Cite Save	Find it@Stanford
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		When standard RANSAC is not enough: cross-media visual matching with hypothesis relevancy <u>T Hassner</u> , L Assif, <u>L Wolf</u> - Machine Vision and Applications, 2013 - Springer Abstract The same scene can be depicted by multiple visual media. For example, the same event can be captured by a comic image or a movie frame; the same object can be represented by a photograph or by a 3D computer graphics model. In order to extract the Related articles Cite Save	[HTML] from springer.com Find it@Stanford
		A real-time system of lane detection and tracking based on optimized RANSAC B-spline fitting J Deng, Y Han - Proceedings of the 2013 Research in Adaptive and, 2013 - dl.acm.org	



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